REFERENCES

[1] R. Bowers, "Plasmas in solids," *Sci. Am.*, vol. 209, pp. 46–53, November 1963.
[2] R. D. Larrabee and W. A. Hicinbothem, Jr., "Observation of microwave emission from indium antimonide," *Proc. of Symp. on Plasma Effects in Solids*. Paris: Dunod, 1965, pp. 181–187. (Symposium held in connection with 7th Internat'l Conf. on Physics of Semiconductors, Paris, France, 1964.)
[3] K. Suzuki, "The generation of microwave radiation from InSb," *Japan. J. Appl. Phys.*, vol. 4, pp. 42–52, January 1965.
[4] T. Suzuki, "Microwave emission and low frequency instabilities in InSb," *Japan. J. Appl. Phys.*, vol. 4, pp. 700–701, September 1965.
[5] B. Ancker-Johnson and M. F. Berg, "Microwave emission from *P*-type indium antimonide," *Bull. Am. Phys. Soc.*, ser. II, vol. 10, p. 720, September 1965.
[6] M. Toda, "Saturation of the microwave impedance and the occurrence of microwave emission in InSb," *Japan. J. Appl. Phys.*, vol. 4, pp. 854–860, November 1965.
[7] B. Ancker-Johnson and M. F. Berg, "Microwave radiation from electron-hole plasmas," presented at 7th Ann. Meeting of Division of Plasma Physics, American Physical Society, San Francisco, Calif., November 1965; page S7 of *Program and Abstracts* of papers.
[8] R. D. Larrabee and W. A. Hicinbothem, Jr., "Current oscillations and microwave emission in indium antimonide," *IEEE Trans. on Electron Devices*, vol. ED-13, pp. 121–131, January 1966.
[9] K. Suzuki, "The spectrum of microwave radiation from InSb," *IEEE Trans. on Electron Devices*, vol. ED-13, pp. 132–136, January 1966.
[10] H. Morisaki and Y. Inuishi, "Harmonic generation in microwave emission from InSb," *Japan. J. Appl. Phys.*, vol. 5, pp. 343–344, April 1966.
[11] ——, "Hall mobility and microwave emission in InSb plasma," *Japan. J. Appl. Phys.*, vol. 5, p. 637, July 1966.
[12] R. Bowers and M. C. Steele, "Plasma effects in solids," *Proc. IEEE*, vol. 52, pp. 1105–1112, October 1964.
[13] M. C. Steele, "Possible explanation for microwave emission from InSb in magnetic fields," *Proc. of Symp. on Plasma Effects in Solids*. Paris: Dunod, 1965, pp. 189–191. (Symposium held in connection with 7th Internat'l Conf. on Physics of Semiconductors, Paris, France, 1964.)
[14] B. Vural and M. C. Steele "Possible two-stream instabilities of drifted electron-hole plasmas in longitudinal magnetic fields," *Phys. Rev.*, vol. 139, pp. A300–A304, July 1965.
[15] M. Glicksman and W. A. Hicinbothem, Jr., "Hot electrons in indium antimonide," *Phys. Rev.*, vol. 129, pp. 1572–1577, February 1963.
[16] M. C. Steele, "Microwave generation from photoconductive mixing of amplified spontaneous radiation," *RCA Rev.*, vol. 27, pp. 263–271, June 1966.
[17] S. J. Buchsbaum, A. G. Chynoweth, and W. L. Feldman, "Microwave emission from indium antimonide," *Appl. Phys. Lett.*, vol. 6, pp. 67–69, February 1965.


## Perturbations of the Critical Parameters of Quarter-Wave Directional Couplers

The theory of coupled transmission lines was discussed by Jones and Bolljahn[1] on the assumption that the phase velocities for even and odd modes $v_e$ and $v_o$ were equal to each other. However, when the coupled lines are constructed in the microstrip geometry or on a hard substrate suspended between two ground planes, the phase velocities are in general unequal. As a result, the even- and odd-mode characteristic impedances $Z_{0e}$ and $Z_{0o}$, as well as the phase velocities, must be adjusted to prescribed values. However, little is known how critical these adjustments are in order to get desired coupler performance.

This correspondence reports some results obtained for the scattering coefficients $S_{ij}$ of 3-, 10-, and 20-dB quarter-wave couplers.

For an ideal coupler, $S_{11}=S_{31}=0$ independent of frequency, and $S_{21}$ and $S_{41}$ have their maximum and minimum values at the center frequency $\omega_0$, respectively, where the subscripts refer to port numbers as shown in Fig. 1.

To set up the equations for the scattering coefficients we shall make use of the double symmetry of the two parallel lines. Energy from a generator with a voltage 4E and a source impedance $Z_0$ will be fed into port 1 in four steps and then superimposed. This is shown in Fig. 1. The first two steps represent even-mode excitation, the last two steps odd-mode excitation. The electrical lengths of the coupling section with physical length $l$ are $2\theta_e=\omega l/v_e$ and $2\theta_o=\omega l/v_o$, respectively.

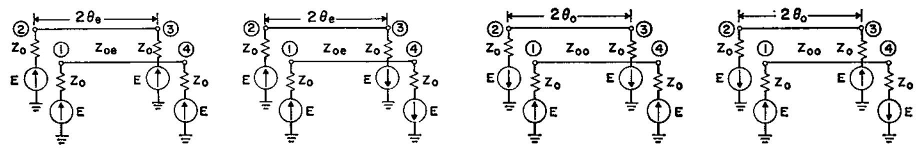
Because of the polarity of the generator voltages, the current is zero at the center of the lines for the configuration in Figs. 1(a) and 1(c), and the voltage is zero for Figs. 1(b) and 1(d). Therefore, at the center, the lines may be open or short-circuited, respectively, without changing the current at each port. The currents into port 1 for the four cases are therefore given by

$$i_1 = \frac{E}{-jZ_{0e}\cot\theta_e + Z_0}$$

$$i_2 = \frac{E}{jZ_{0e}\tan\theta_e + Z_0}$$

$$i_3 = \frac{E}{-jZ_{0o}\cot\theta_0 + Z_0}$$

$$i_4 = \frac{E}{jZ_{0o}\tan\theta_0 + Z_0}\cdot \qquad (1)$$



(a) (b) (c) (d)

(a) and (b) Two states of even-mode excitation of the coupler.

(c) and (d) Two states of odd-mode excitation of the coupler.

Fig. 1.

TABLE I

| | | 10% Change in $v_e$ or $v_o$ | 10% Change in $Z_{0e}$ or $Z_{0o}$ |
|---|---|---|---|
| 3-dB Coupler | Return loss (VSWR) | 28 dB (1.08) | 32 dB (1.05) |
| | Directivity | 25 dB | 29 dB |
| 10-dB Coupler | Return loss (VSWR) | 33 dB (1.04) | 27 dB (1.09) |
| | Directivity | 13 dB | 26 dB |
| 20-dB Coupler | Return loss (VSWR) | 42 dB (1.02) | 26 dB (1.1) |
| | Directivity | 2 dB | 26 dB |

TABLE II

| | Type of Coupler | Permissible $v_e$ or $v_o$ | Change in $Z_{0e}$ or $Z_{0o}$ |
|---|---|---|---|
| For 1.05 input VSWR | 3 dB | 6.5% | 10.0% |
| | 10 dB | 11.0% | 5.5% |
| | 20 dB | +45% / −25% * | 5.5% |
| For 30-dB directivity | 3 dB | 6.0% | 9.0% |
| | 10 dB | 1.5% | 6.5% |
| | 20 dB | 0.4% | 6.5% |

* These values were calculated from the exact equation for $S_{11}$.

Manuscript received February 6, 1967.

[1] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-4, pp. 75–81, April 1956.

By superposition and proper permutation of the signs of these four currents the actual coupler currents at the four ports are obtained and the scattering coefficients can be calculated as follows:

$$S_{11} = 1 - \frac{Z_0}{2E}(i_1 + i_2 + i_3 + i_4),$$

$$S_{21} = -\frac{Z_0}{2E}(i_1 + i_2 - i_3 - i_4)$$

$$S_{31} = -\frac{Z_0}{2E}(i_1 - i_2 - i_3 + i_4),$$

$$S_{41} = -\frac{Z_0}{2E}(i_1 - i_2 + i_3 - i_4). \qquad (2)$$

From these scattering coefficients all necessary information can be derived. For example, for small variations of $v_e$, $v_o$, $Z_{0e}$, and $Z_{0o}$ from their ideal values, the first-order approximation of the scattering coefficients at $\omega_0$ can be obtained as follows:

$$S_{11} = \frac{1}{2}\left(\frac{\Delta Z_{0e}}{Z_{0e}} + \frac{\Delta Z_{0o}}{Z_{0o}}\right)(1 - k^2) + j\frac{\pi}{4}\left(\frac{\Delta v_e}{v_e} - \frac{\Delta v_o}{v_o}\right)k\sqrt{1 - k^2}$$

$$S_{21} = k + \frac{1}{2}\left(\frac{\Delta Z_{0e}}{Z_{0e}} - \frac{\Delta Z_{0o}}{Z_{0o}}\right)(1 - k^2) + j\frac{\pi}{4}\left(\frac{\Delta v_e}{v_e} + \frac{\Delta v_o}{v_o}\right)k\sqrt{1 - k^2}$$

$$S_{31} = \frac{\pi}{4}\left(\frac{\Delta v_e}{v_e} - \frac{\Delta v_o}{v_o}\right)(1 - k^2) + \frac{j}{2}\left(\frac{\Delta Z_{0e}}{Z_{0e}} + \frac{\Delta Z_{0o}}{Z_{0o}}\right)k\sqrt{1 - k^2}$$

$$S_{41} = -j\sqrt{1 - k^2} + \frac{\pi}{4}\left(\frac{\Delta v_e}{v_e} + \frac{\Delta v_o}{v_o}\right)(1 - k^2) + \frac{j}{2}\left(\frac{\Delta Z_{0e}}{Z_{0e}} - \frac{\Delta Z_{0o}}{Z_{0o}}\right)k\sqrt{1 - k^2} \qquad (3)$$

where

$$k = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} = |S_{21}|. \qquad (4)$$

These equations are useful for parameter variations between 0 and 10 percent.

The perturbation of the coupling $S_{21}$ and $S_{41}$ caused by changes in $v_e$ or $v_o$ results mainly in a change of the phases of $S_{21}$ and $S_{41}$ rather than of their amplitude, but in such a way that the 90° phase difference between the phases of the waves leaving the two ports is maintained. The perturbation of $S_{21}$ and $S_{41}$ caused by a variation of $v_e$ may be cancelled by the opposite variation of $v_o$.

An increase in $Z_{0e}$ increases the amplitude of $S_{21}$ and hence decreases $S_{41}$, according to (4). Table I shows the return loss (and corresponding VSWR) and the directivity (i.e., the difference of the output levels between port 3 and port 2) for 10-percent variations in impedance or velocity for a 3-, 10-, and 20-dB coupler.

Table II contains the permissible percentage of variation of any of the $v$ or $Z$ parameters for practical values of an input VSWR of 1.05 and a directivity of 30 dB.

In general, the following applies: The smaller the coupling factor $k$, the smaller is the absolute value of the perturbation of $S_{11}$ and $S_{21}$ and the larger it is for $S_{31}$ and $S_{41}$ for a given variation of $v_e$ or $v_o$. Thus, for the 20-dB coupler, velocity perturbations have a strong effect on directivity and very little effect on input VSWR. The situation is different for the perturbation of the characteristic impedances: The effects on $S_{11}$ and $S_{21}$ are now larger for couplers with smaller $k$. Consequently, the input VSWR gets larger for decreasing $k$ for a given impedance variation. On the other hand, the perturbations of $S_{31}$ and $S_{41}$ become smaller for decreasing $k$ but $S_{31}$ does not decrease fast enough to improve the directivity. Therefore, the directivity also becomes worse with decreasing $k$. The input VSWR and the directivity have almost the same values for the 10-dB coupler and the 20-dB coupler for a given impedance perturbation.


## Acknowledgment

Acknowledgment is due K. Kurokawa and E. A. J. Marcatili for discussions and comments made during the preparation of this correspondence.



H. E. Brenner
Bell Telephone Labs., Inc.
Murray Hill, N. J.